A ReaDiNet seminar day on reaction-diffusion systems in biology

November 21st, 2022 - Nancy

Room Doblin - IECL

Monday, November 21st

- **10:00 - 10:50 : Harunori Monobe**, Osaka Metropolitan University
  Compact traveling waves for a mean-curvature flow with driving force
- **10:50 - 11:40: Romain Ducasse**, Université Paris Cité
  A cross-diffusion system obtained via (convex) relaxation in the JKO scheme
- **11:40 - 12:20: Léo Girardin**, CNRS & Université Claude Bernard Lyon-1
  Non-local pulling in reaction–diffusion equations

Lunch break

- **14:00 - 14:50 : Hirofumi Izuhara**, University of Miyazaki
  Cross-diffusion in predator-prey models with two behavioral states in predators
- **14:50 - 15:40 : Matthieu Alfaro**, Université de Rouen-Normandie
  Adaptation in a heterogeneous environment: to be three or not to be

Coffee break

- **16:00 - 16:50 : Mingmin Zhang**, Institut de Mathématiques de Toulouse
  Propagation and blocking in a two-patch reaction-diffusion model

Matthieu Alfaro

Adaptation in a heterogeneous environment: to be three or not to be

Romain Ducasse

A cross-diffusion system obtained via (convex) relaxation in the JKO scheme

In this talk, we start from a very natural system of cross-diffusion equations, which can be seen as a gradient flow for the Wasserstein distance of a certain functional. Unfortunately, the cross-diffusion system is not well-posed, as a consequence of the fact that the underlying functional is not lower semi-continuous. We then consider the relaxation of the functional, and prove existence of a solution in a suitable sense for the gradient flow of (the relaxed functional). This gradient flow has also a cross-diffusion structure, but the mixture between two different regimes, that are determined by the relaxation, makes this study non-trivial. This is joint work with F. Santambrogio and H. Yoldas.

Léo Girardin

Non-local pulling in reaction–diffusion equations

In recent years it has been established that, in reaction–diffusion models of monostable type, a favorable environment that flees away from a population density arising from a localized initial condition can in fact improve the spreading speed of this population density. In a simplified Fisher–KPP setting, when the speed of the favorable area is sufficiently small for the population to keep up, then the spreading speed is correctly predicted by the Fisher–KPP speed in this environment. But when the speed is too large for the population to keep up, the Fisher–KPP speed of the second, less favorable environment, is in general only a lower estimate for the spreading speed: the population can be "non-locally pulled" by its exponential tail in the favorable area, even though the distance between this area and the spreading front is linearly increasing in time. The exact spreading speed is given by an explicit formula and at least two methods of proof are known. In this talk I will review some results about non-local pulling in reaction–diffusion systems and equations and then I will present a work in progress in collaboration with Thomas Giletti and Hiroshi Matano.

Hirofumi Izuhara

Cross-diffusion in predator-prey models with two behavioral states in predators

Cross-diffusion may be an important driving force of pattern formation in population models. Recently, a relation between cross-diffusion and reaction-diffusion systems has been revealed from the mathematical modeling point of view. In this talk, we derive a predator-prey model with cross-diffusion from a simple reaction-diffusion system with two behavioral states in the predator population and examine whether cross-diffusion can induce spatial patterns in predator-prey models. A simple situation assumes that the predators of two states have identical behavioral characteristics except for their mobility. Our analysis shows that cross-diffusion derived in this situation can induce spatial patterns if the prey-density-dependent conversion rate from mobile to immobile increases at low prey density and decreases at high prey density.

Harunori Monobe

Compact traveling waves for a mean-curvature flow with driving force

Mean-curvature flow with a driving force appears in various mathematical problems such as motion of soap films, grain boundaries and singular limit problems of various reaction diffusion systems, e.g., FitzHugh-Nagumo equation. In this talk, we show the existence and uniqueness of traveling waves, composed of a Jordan curve (or closed surface), for an anisotropic curvature flow with a driving force. We call such a solution "compact traveling wave" in this talk. Our aim is to investigate the condition of external driving force when the curvature flow has traveling waves. This is a joint work with Professor Hirokazu Ninomiya (Meiji University).

Mingmin Zhang

Propagation and blocking in a two-patch reaction-diffusion model

This talk is concerned with propagation phenomena for the solutions of the Cauchy problem associated with a two-patch one-dimensional reaction-diffusion model. It is assumed that each patch has a relatively well-defined structure which is considered as homogeneous. A coupling interface condition between the two patches is involved. We first study the spreading properties of solutions in the case when the per capita growth rate in each patch is maximal at low densities, a configuration which we call the KPP-KPP case, and which turns out to have some analogies with the homogeneous KPP equation in the whole line. Then, in the KPP-bistable case, we provide various conditions under which the solutions show different dynamics in the bistable patch, that is, blocking, virtual blocking (propagation with speed zero), or spreading with positive speed. Moreover, when propagation occurs with positive speed, a global stability result is proved. Finally, the analysis in the KPP-bistable frame is extended to the bistable-bistable case. This is based on joint work with Prof. Francois Hamel and Prof. Frithjof Lutscher.